# Binary Adders <br> Lecture 6 Section 2.5 

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(9) Binary Addition

(2) Half Adders

(3) Full Adders

4 Assignment

## Outline

(9) Binary Addition

## 2 Half Adders

## (3) Full Adders

## 4) Assignment

## Binary Adders

- A half adder (HA) adds two bits and produces a sum bit and a carry bit.
- A full adder (FA) adds two bits and a carry-in bit and produces a sum bit and a carry-out bit.


## Outline

(1) Binary Addition

(2) Half Adders

## (3) Full Adders

4 Assignment

## A Half Adder

| Input |  | Output |  |
| :---: | :---: | :---: | :---: |
| $p$ | $q$ | sum | carry |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 |

- Addition of binary digits by a half adder.


## A Half Adder

- We see that
- The sum bit is $p \oplus q$.
- The carry bit is $p \wedge q$.
- Design circuits for $p \oplus q$ and $p \wedge q$.
- Combine them into a single circuit.


## Outline

## (1) Binary Addition

## (2) Half Adders

(3) Full Adders

## 4 Assignment

## A Full Adder

| Input |  |  | Output |  |
| :---: | :---: | :---: | :---: | :---: |
| $p$ | $q$ | carry-in | sum | carry-out |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |

- Addition of binary digits by a full adder.


## A Full Adder

- We can express the sum bit as

$$
(p \oplus q) \oplus c_{\mathrm{in}}=p \oplus q \oplus c_{\mathrm{in}}
$$

and the carry-out $c_{\text {out }}$ as

$$
(p \wedge q) \vee\left((p \oplus q) \wedge c_{\mathrm{in}}\right)
$$

- We can implement these two expressions in a circuit.


## A Full Adder

- Use a HA to add $p$ and $q$. This produces

$$
\begin{aligned}
& s^{\prime}=p \oplus q . \\
& c^{\prime}=p \wedge q .
\end{aligned}
$$

- Then use another HA to add $c_{\text {in }}$ to $s^{\prime}$ to get the final sum and carry-out bits.

$$
\begin{aligned}
s & =s^{\prime} \oplus c_{\mathrm{in}} . \\
c_{\mathrm{out}} & =c^{\prime} \vee\left(s^{\prime} \wedge c_{\mathrm{in}}\right) .
\end{aligned}
$$

## Circuits



## Circuits

- Design a circuit for a full adder using two half adders.
- Design a circuit for an 8-bit adder using 8 full adders.


## Outline

## (1) Binary Addition

(2) Half Adders
(3) Full Adders
4) Assignment

## Assignment

## Assignment

- Read Section 2.5, pages 78-84.
- Exercises 1, 2, 7, 8, 13, 14, 17, 18, 21, page 94.

